Buckling of Plates

UNIT - IV

Rectangular sheets under compression, local buckling stress of thin walled sections, crippling stresses by Needham's and Gerard's Methods. Thin walled column Strength. Sheet stiffener panels Effective Width, inter rivet and sheet wrinkling failures

Buckling:

Buckling is defined as large lateral finite deformation (bowing, warping, wrinkling or twisting) due to axial compressive loads (or stresses) occurring throughout a structure (or) some portion thereof.

Primary buckling:

* Primary buckling, otherwise known as global buckling, occurs in long columns and in sheet - stiffener panels having large thickness.
* Primary buckling deformation extends over the major dimensions of a structure ie the entire unit buckles as a whole.
* Under sustained loading, primary buckling usually results in significant loss of structural integrity.

Secondary buckling:

* Secondary buckling, otherwise known as local buckling, is confined to localized regions, such as the cross sections of individual members. It occurs in sheet - stiffener panels of smaller thickness.
* The skin buckles between the stiffeners which the stiffener is still capable of resisting compressive loads.
* Secondary buckling may cause some elements of the structure to fail, resulting in a re-distribution of stress. This may (or) may not

degrade structural integrity.

Rectangular sheet under compression:

The equation of buckling stress under compression for rectangular thin sheet is given by

$$\sigma_{cr} = \frac{\pi^2 K_c E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2$$

where

K_c - buckling co-efficient which depends on the boundary condition and $\left(\frac{a}{b}\right)$ ratio is sheet aspect ratio.

E = Modulus of Elasticity

ν = Elastic poisson's ratio.

b = short dimension of plate

t = sheet thickness.

If the buckling occur @ stress in the inelastic stress range than E and ν are not same as the elastic buckling and hence plastic buckling correction factor η is included

$$\sigma_{cr} = \frac{\eta \pi^2 K_c E}{12(1-\nu^2)} \left[\frac{t}{b}\right]^2$$

where η is plasticity reduction factor and equal to σ_{cr} Plastic / σ_{cr} elastic.

(or) $\eta = \frac{E_{sec}}{E} = \frac{E_s}{E}$

"η" measures the stiffness of the material in inelastic region and hence stress strain is non-linear.

Al alloy sheet are available with thin covering of Pure aluminium Such material refered as cad aluminium alloy

The mechanical strength of clad - material is lower than core material and clad are loaded to extreme - because of located @ extreme of thickness and hence extreme strain

The correction must made because of lower strength of clad material called clad correction factor and

$$\overline{\sigma_{cr}} = \eta \, \overline{\sigma_{cr}}$$

Buckling of plate under shear load :

The shear buckling stress of the rectangular plate under elastic load condition is given by

$$\tau_{cr} = \frac{\pi^2 K_s E}{12(1-\gamma^2)} \left(\frac{t}{b}\right)^2$$

where, K_s = Shear Buckling co-efficient

b = Short dimension of plate

γ = poission's ratio

then τ_{cr} is a function of plate aspect ratio $\left(\frac{a}{b}\right)$ and boundary condition.

If the buckling occurs at stress level above the elastic stress than plasticity correction factor must be included and buckling stress equation under plastic condition given by.

$$\tau_{cr} = \frac{\eta_s \pi^2 K_s E}{12(1-\gamma^2)} \left(\frac{t}{b}\right)^2$$

where η_s = ...

$$\eta_s = \frac{G_s}{G}$$

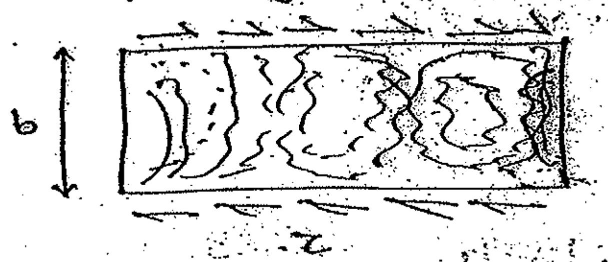
where G_s = Shear Secant Modulus

G = Shear Modulus.

which are obtained from Shear & Stress-Strain relation of material the material which are loaded

η_c Gives the stiffness of in plastic Strain condition.

A long rectangular plate Subjected to pure Shear produces internal compressive Stress on planes at 45° with plate edges and thus these compressive causes long panel to buckle in patterns at an angle to plate edges shown in figure.



Buckling of plates under bending load.

The equation of buckling of plate under bending loads is Same as for the compression & Shear except the buckling co-efficient K_b is different from K_c and K_s.

When a plate buckles under bending then it involves relatively Short wave length equal to $2/3\,b$ for long simply Supported edges and Smaller buckling Pattern (ie wave length) result is the K_b is too larger than K_c (or) K_s

The buckling Stress Equation becomes

$$\sigma_{cr} = \frac{\pi^2 K_b E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$

For bending inelastic buckling

$$\sigma_{cr} = \frac{\pi^2 \eta_b K_b E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$

where K_b = buckling co-efficient of bending depends on $\left(\frac{a}{b}\right)$ ratio and edge restraint (t) against rotation.

η_b = plastic reducing factor of bending

fn of $\eta = \frac{E_{sec}}{E}$

Combined longitudinal and compression

$$R_b^{1.75} + R_c = 1$$

Combined bending and Shear

$$R_b^2 + R_s^2 = 1$$

Margin of Safety

$$M_s = 1/R_b^2 + R_s^2 - 1$$; R_s = Stress ratio.

Combined Shear and longitudinal compression

$$R_s^2 + R_L = 1$$

$$M_s = \frac{2}{CR_L + \sqrt{R_L^2 + 4R_s^2}}$$

[5]

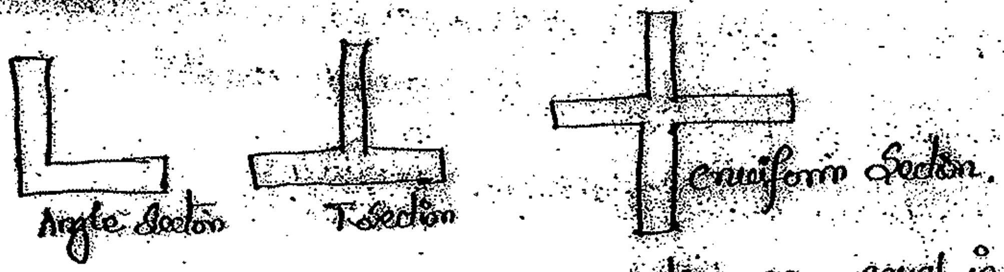
* Thin flat sheet is inefficient for carrying compressive loads because the buckling stress are relatively low.

* This weakness fault can be greatly improved by forming the flat sheet in composite shapes such as angles, channels and zeas.

* Most of the composite shape can also made by extruding process.

Compressive buckling stress for equal flanged elements:

The simplest equal flanged member that can be formed is the angle shape. Other shapes with equal flanges are T section and cruciform section.



The flanges which makeup the section are equal in size, each flange will buckle at the same stress. The flange cannot restrain the other.

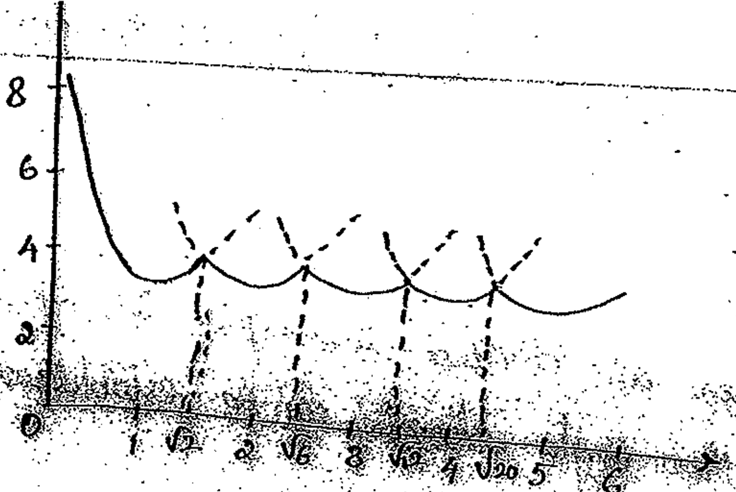
Buckling compressive stress for a long flange

$$\sigma_{cr} = \frac{\pi^2 K_c E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2$$

where the plate buckling co-efficient K is given by the minimum of

$$K = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2$$

The minimum value of K is obtained from the lower envelope of curves. Different values of m as shown by dotted curves.



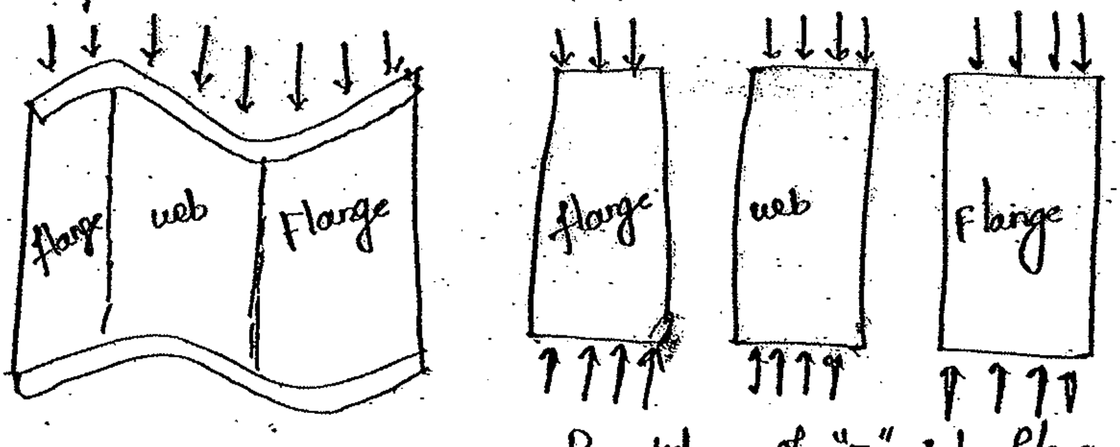
m Varies with the ratio a/b and that k & the buckling loads are a minimum when k = 4 and Values of a/b = 1,2,3 --- k is very nearly constant for $\frac{a}{b} > 3$. This factor is particularly useful in a/c Structures where longitudinal Stiffeners are used to divide the Skin into narrow panels.

Compressive buckling Stress for Simple web flange elements :

The most common flange web Structural Shapes are channels, zees and hat Sections.

In Z Section, two flanges and 1 web, the buckling strength depends on the boundary restraint between the two elements.

If this restraint, which is unknown could be found interms of rotational restraint (t).

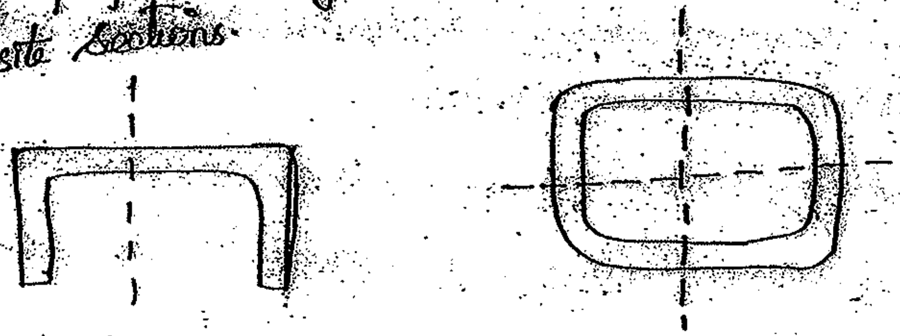


Breakdown of "Z" into flange & web elements

Local buckling and local failure loads are not the same. Local buckling at low stresses, whereas crippling (or) failure occurs at high stresses.

Needham's Angle method

1. Member sections are divided into equal (or) unequal angles.
2. Strength of these angle elements can be established by either theory (or) test.
3. Ultimate failing strength of the angle section can be obtained by adding up of the strength of the angle elements that make up the composite sections.



By Needham's angle method,

$$\frac{F_{cs}}{(F_{cy}.E)^{1/2}} = \frac{C_e}{\left(\frac{b'}{t}\right)^{0.75}}$$

where
F_{cs} = crippling stress
F_{cy} = yield stress
$\frac{b'}{t} = \frac{a+b}{2t}$

C_e = elasticity coefficient

C_e = 0.316 for two edges free
= 0.342 for one edge free
= 0.366 for No edge free.

$$\text{If} \; g \ldots, \quad P_{cs} = F_{cs} \times A$$

Finally, $F_{cs} = \dfrac{\Sigma P_{cs}}{\Sigma A}$.

Gerard's Method to find crippling stress:

It is the wider application of Needham's method. It is also known as Semi-emperical method to determine the crippling stress. For sections with distorted unloaded edges such as angles, V-groove, multi-corner section, the equation is (with ± 10% limits)

$$\frac{F_{cs}}{F_{cy}} = 0.56 \left[\left(\frac{gt^2}{A}\right) * \left(\frac{E}{F_{cy}}\right)^{1/2} \right]^{0.85}$$

For T, +, H sections:- (with ± 5% limits)

$$\frac{F_{cs}}{F_{cy}} = 0.67 \left[\left(\frac{gt^2}{A}\right) * \left(\frac{E}{F_{cy}}\right)^{1/3} \right]^{0.40}$$

For 2-corner, Z, J, [sections

$$\frac{F_{cs}}{F_{cy}} = 3.2 \left[\left(\frac{t^2}{A}\right) * \left(\frac{E}{F_{cy}}\right)^{1/3} \right]^{0.75}$$

where

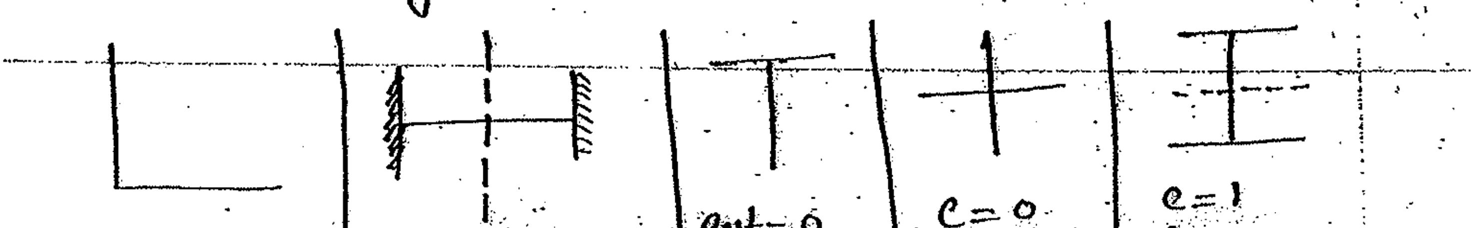
F_{cs} = crippling stress

F_{cy} = yield stress

t = thickness of the element.

A = cross sectional Area.

g = No of cuts ~~been~~ made + No of flanges.



Cut = 0	Cut = 0	Cut = 0	c = 0	c = 1
flange = 2	flange = 2	f = 3	f = 4	f = 6
g = 0+2 = 2	g = 0+2 = 2	g = 3	g = 4	g = 7

Failure by Inter - Rivet Buckling:

The effective sheet area is considered to act monolithically with stiffeners, however if the rivets are spot welded, that fasten the rivet to the stiffener are spaced too far apart, sheet will buckle before the crippling stress of the stringer is placed. In order to prevent this type of buckling, rivet spacing has to be selected on the upper surface. Rivet spacing should be closer than the lower surface because the compressive loads acts on the top of the wing.

crippling stress for inter rivet buckling can be obtained as follows

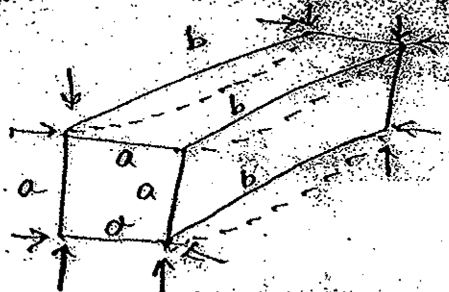
$$F_{cr} = \frac{\pi^2 E_t}{\left(P/0.58t\right)^2}$$

Failure by Rivet wrinkling:

* If the rivet space is relatively large then the sheet will start buckle between the rivets. This buckling does not deform in flange to which the sheet is attached.
* The rivets spacing is such that to prevent the inter rivet buckling then the failure of the sheets will occur by wrinkling. It is also known as poster wrinkling of the riveted panel.

Primary (or) general instability buckled mode shapes have wave lengths in the order of the length of the column itself and there is no deflection in the plane of the cross section.

If column is thin walled, we can be concerned with secondary (or) local instability buckling of the individual thin walled components of the column construction.



The average compressive stress on the cross-section of the failure load is called crippling stress. The figure shows a compression member with a thin walled closed cross-section. Each wall is a thin plate of large aspect ratio. If we consider each of narrow plate, it is to be simply supported along their sides then we might expect the buckling of the type illustrated as shown in fig. To calculate the crippling stress of a column with "n" walls of the cross section are idealized in rectangles. consider the i^{th} wall having the thickness t_i and length b_i. These rectangles are considered to be simply supported. plates with common edge for the considered segments to be simply supported along that side. The average crippling stress σ_{cc} in the i^{th} wall is denoted by

σ_{cci}^{th} is found by the emperical formula

$$\frac{\sigma_{cc}}{\sigma_{cy}} = \alpha \left[\frac{\pi^2 c}{12(1-\nu^2)} \times \frac{1}{\lambda^2} \right]^{1-n}$$

where $\lambda = \sqrt{\frac{\sigma_{cy}}{E}} \times \frac{b}{t}$

σ_{cy} = Compressive yield stress

c = plate constant → coefficient for elastic buckling

α, n = Experimentally determined parameters.

Commonly, $\alpha = 0.8$, $n = 0.8$, $\nu = 0.3$

for no edge free, $c = 4.0$

for one edge free ; $c = 0.425$

So $\frac{\sigma_{cc}}{\sigma_{cy}} = 1.34\ \lambda^{-0.8}$ [No edge free]

$\frac{\sigma_{cc}}{\sigma_{cy}} = 0.546\ \lambda^{-0.8}$ [one edge free]

* If the calculated crippling stress for the given wall exceeds the cut off stress (σ_{co}) for the wall material, then the value of σ_{co}, is taken to be the crippling stress.
* For many alloys and metals the cut off stress (σ_{co}) for compressive buckling is just the compressive yield strength (σ_{cy})
* cut off stresses for compressive buckling of unstiffened flat plates are given below.

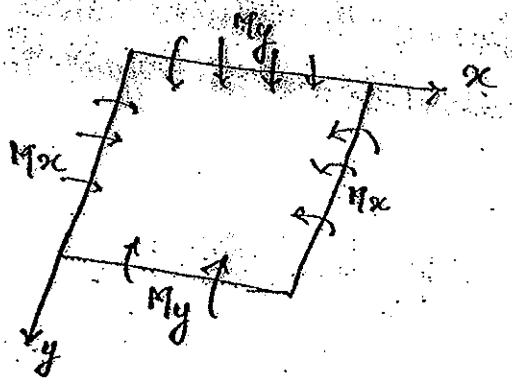
<u>Material</u> →	<u>cut off stress</u>
Al 2014-T, 2024-T, 6061-T	$\sigma_{cy}\left[1 + \frac{\sigma_{cy}}{200,000}\right]$
Al 7075-T	$1.075 \times \sigma_{cy}$
Laminated composites	$1.1 \times \sigma_{cy}$
other Material	σ_{cy}

...ss Section, we find the ... $\sigma_{cc}(i)$ for each wall of the average crippling Stress for the column Cross-Section using the formula

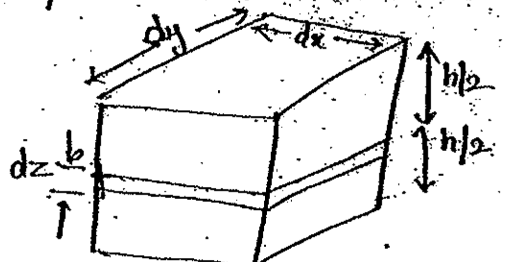
$$\sigma_{cc} = \frac{\sum_{i=1}^{n} \sigma_{cc}(i) A_i}{\sum_{i=1}^{i=n} A_i} \qquad \text{where } A_i = b_i \times t_i$$

For all failures, crippling Stress in the corner region of the cross-section is slightly to be higher than the walls. So the average crippling Stress will be greater than the one given by above eqn. So on account of this, we ignore the corner material in computing each wall length b_i.

<u>Buckling Stress of a Rectangular plate:</u>



Bending Stress of this plate:- Bending moment per unit length of the edge parallel to the x-axis is M_x and ||el to y axis is M_y. These moments are positive when they produce compression at the upper Surface of the plate and tension at the lower Surface of the plate



Thickness is small when compared to the other dimensions. $\frac{1}{R_x}$ and

R_y are the radius of curvatures of neutral surface in sections parallel to xz and yz planes. Unit elongations in the x and y directions of an elemental lamina at a distance of z from the neutral surface ∴ $\varepsilon_x = z/R_x$; $\varepsilon_y = z/R_y \rightarrow (1)$

WKT $\varepsilon_x = \frac{1}{E}\left(\sigma_x - \gamma\sigma_y\right) \rightarrow (2)$

$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \gamma\sigma_x\right) \rightarrow (3)$

Substituting (2) & (3) in (1)

$$\sigma_x = \frac{EZ}{(1-\gamma^2)}\left[\frac{1}{R_x} + \gamma\frac{1}{R_y}\right] \rightarrow (4)$$

$$\sigma_y = \frac{EZ}{(1-\gamma^2)}\left[\frac{1}{R_y} + \gamma\frac{1}{R_x}\right] \rightarrow (5)$$

The normal stresses distributed over the lateral sides of the element can be reduced to couples which must be equal to the external moment

$$M_x \cdot dy = \int_{-h/2}^{h/2} \sigma_x . z . dy . dz \rightarrow (6)$$

$$M_y\, dx = \int_{-h/2}^{h/2} \sigma_y\, z\, dx\, dz \rightarrow (7)$$

Now substitute (4) & (5) in (6) & (7)

$$M_x = \int_{-h/2}^{h/2} \frac{EZ^2}{(1-\gamma^2)}\left(\frac{1}{R_x} + \frac{\gamma}{R_y}\right) dz$$

$$M_y = \int_{-h/2}^{h/2} \frac{EZ^2}{(1-\gamma^2)}\left(\frac{1}{R_x} + \frac{\gamma}{R_y}\right) dz$$

$$\int_{-h/2} \frac{\quad}{1-\nu^2} \quad \frac{\quad}{12(1-\nu^2)}$$

Then $$M_x = D\left[\frac{1}{R_x} + \frac{\nu}{R_y}\right] \longrightarrow (8)$$

$$M_y = D\left[\frac{1}{R_y} + \frac{\nu}{R_y}\right] \longrightarrow (9)$$

where D = flexural Rigidity of the plate.

If "w" is the deflection of any point on the plate in the "z" direction, then we may relate w to the curvature of the plate in the same manner as the well-known expression for beam curvature

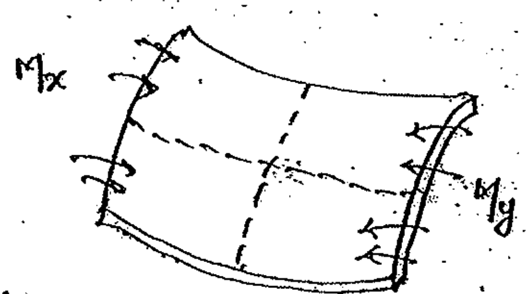
$$\frac{1}{R_x} = -\frac{\partial^2 w}{\partial x^2} \; ; \; \frac{1}{R_y} = \frac{\partial^2 w}{\partial y^2}$$

The negative sign resulting from the fact that the centers of curvature occur above the plate, in which region z is negative.

Equation (8) and (9) becomes

$$M_x = -D\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right] \rightarrow (10)$$

$$M_y = -D\left[\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right] \rightarrow (11)$$



The above equation define the deflected shape of the plate provided that M_x and M_y are known. If either M_x (or) M_y is zero, then

$$\frac{\partial^2 w}{\partial x^2} = -\gamma \frac{\partial^2 w}{\partial y^2} \quad (\text{or}) \quad \frac{\partial^2 w}{\partial y^2} = -\gamma \frac{\partial^2 w}{\partial x^2}$$

and the plate has curvature of opposite signs. A surface possessing two curvatures of opposite sign is known as a anticlastic surface, as opposed to a synclastic surface, which has curvatures of the same sign. Further, if $M_x = M_y = M$, then from equations (10) and (11),

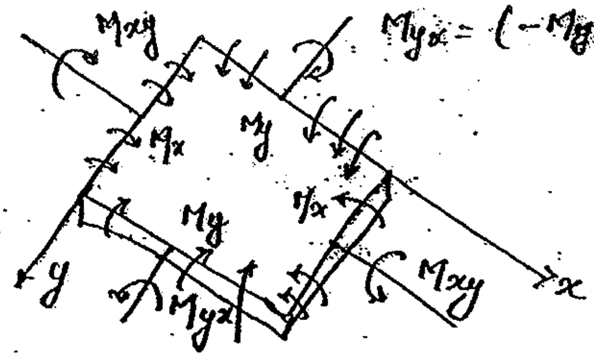
$$\frac{1}{R_x} = \frac{1}{R_y} = \frac{1}{R}$$

∴ The deformed shape of the plate is spherical and of curvature

$$\frac{1}{R} = \frac{M}{D(1+\gamma)}$$

Plates subjected to bending and twisting:-

In general, the bending moments applied to the plate will not be in planes perpendicular to its edges. Such bending moments, however, may be resolved in the normal manner into tangential and perpendicular components as shown in figure. The perpendicular components are seen to be M_x and M_y as before, while the tangential components M_{xy} and M_{yx} (again, these are moments per unit length) produce twisting of plate about axes parallel to x and y axes.



Pg (121, 122, 123, 124)

occur above the plate in which region z is negative. Eqs. (5.5) and (5.6) then become

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \tag{5.7}$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) \tag{5.8}$$

Eqs. (5.7) and (5.8) define the deflected shape of the plate provided that M_x and M_y are known. If either M_x or M_y is zero then

$$\frac{\partial^2 w}{\partial x^2} = -\nu\frac{\partial^2 w}{\partial y^2} \quad \text{or} \quad \frac{\partial^2 w}{\partial y^2} = -\nu\frac{\partial^2 w}{\partial x^2}$$

and the plate has curvatures of opposite signs. The case of $M_y = 0$ is illustrated in Fig. 5.3. A surface possessing two curvatures of opposite

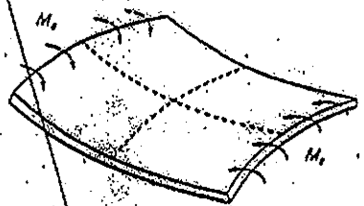


FIG. 5.3 Anticlastic bending.

sign is known as an *anticlastic surface* as opposed to a *synclastic surface* which has curvatures of the same sign. Further if $M_x = M_y = M$ then from Eqs. (5.5) and (5.6)

$$\frac{1}{\rho_x} = \frac{1}{\rho_y} = \frac{1}{\rho}$$

Therefore the deformed shape of the plate is spherical and of curvature

$$\frac{1}{\rho} = \frac{M}{D(1+\nu)} \tag{5.9}$$

5.2 Plates subjected to bending and twisting

In general the bending moments applied to the plate will not be in planes perpendicular to its edges. Such bending moments however may be resolved in the normal manner into tangential and perpendicular components as shown in Fig. 5.4. The perpendicular components are seen to be M_x and M_y as before while the tangential components M_{xy} and M_{yx}

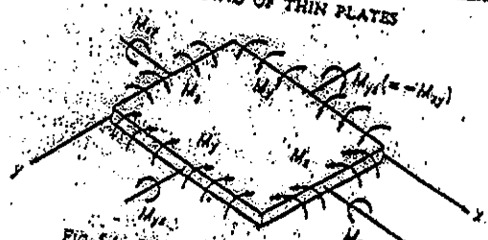


FIG. 5.4 Plate subjected to bending and twisting.

(again these are moments per unit length) produce twisting of the plate about axes parallel to the x and y axes. The system of suffixes and sign convention for these twisting moments must be clearly understood to avoid confusion. M_{xy} is a twisting moment intensity in a vertical x plane parallel to the y axis while M_{yx} is a twisting moment intensity in a vertical y plane parallel to the x axis. Note that the first suffix gives the direction of the axis of the twisting moment. We also define positive twisting moments as being clockwise when viewed along their axes in directions parallel to the positive directions of the corresponding x or y axis. In Fig. 5.4, therefore, all moment intensities are positive.

Since the twisting moments are tangential moments or torques they are resisted by a system of horizontal shear stresses τ_{xy}, as shown in Fig. 5.6. From a consideration of complementary shear stresses (see Fig. 5.6) $M_{xy} = -M_{yx}$, so that we may represent a general moment application to the plate in terms of M_x, M_y and M_{xy} as shown in Fig. 5.5(a). These moments produce tangential and normal moments, M_t and M_n, on an arbitrarily chosen diagonal plane FD. We may express these moment intensities (in an analogous fashion to the complex stress systems of

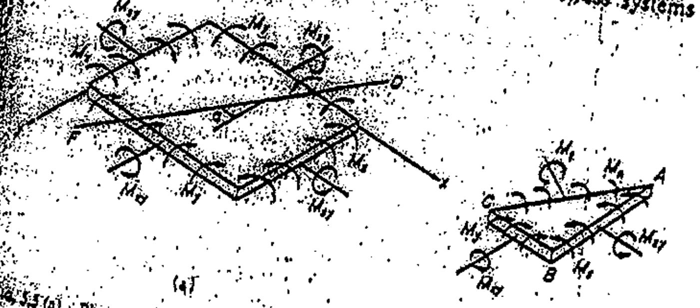


FIG. 5.5 (a) Plate subjected to bending and twisting. (b) Tangential and normal moments on an arbitrary plane.

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Section 1.6) in terms of M_x, M_y and M_{xy}. Thus for equilibrium of the triangular element ABC of Fig. 5.5(b) in a plane perpendicular to AC

$$M_n AC = M_x AB \cos\alpha + M_y BC \sin\alpha - M_{xy} AB \sin\alpha - M_{xy} BC \cos\alpha$$

giving

$$M_n = M_x \cos^2\alpha + M_y \sin^2\alpha - M_{xy} \sin 2\alpha \tag{5.10}$$

Similarly for equilibrium in a plane parallel to CA

$$M_t AC = M_x AB \sin\alpha - M_y BC \cos\alpha + M_{xy} AB \cos\alpha - M_{xy} BC \sin\alpha$$

or
$$M_t = \frac{(M_x - M_y)}{2} \sin 2\alpha + M_{xy} \cos 2\alpha \tag{5.11}$$

(Compare Eqs. (5.10) and (5.11) with Eqs. (1.8) and (1.9)). We observe from Eq. (5.11) that there are two values of α, differing by 90° and given by

$$\tan 2\alpha = -\frac{2M_{xy}}{M_x - M_y}$$

for which $M_t = 0$ leaving normal moments of intensity M_n on two mutually perpendicular planes. These moments are termed *principal moments* and their corresponding curvatures *principal curvatures*. For a plate subjected to pure bending and twisting in which M_x, M_y and M_{xy} are invariable throughout the plate the principal moments are the algebraically greatest and least moments in the plate. It follows that there are no shear stresses on these planes and that the corresponding direct stresses, for a given value of z and moment intensity, are the algebraically greatest and least values of direct stress in the plate.

Let us now return to the loaded plate of Fig. 5.5(a). We have established in Eqs. (5.7) and (5.8), the relationships between the bending moment intensities M_x and M_y and the deflection w of the plate. The next step is to relate the twisting moment M_{xy} to w. From the principle of superposition we may consider M_{xy} acting separately from M_x and M_y. As stated previously M_{xy} is resisted by a system of horizontal complementary shear stresses on the vertical faces of sections taken throughout the thickness of the plate parallel to the x and y axes. Consider an element of the plate formed by such sections as shown in Fig. 5.6. The complementary shear stresses on a lamina of the element a distance z below the neutral plane are, in accordance with the sign convention of Section 1.2, τ_{xy}. Therefore on the face $ABCD$

$$M_{xy}\,\delta y = -\int_{-t/2}^{t/2} \tau_{xy}\,\delta y\, z\, dz$$

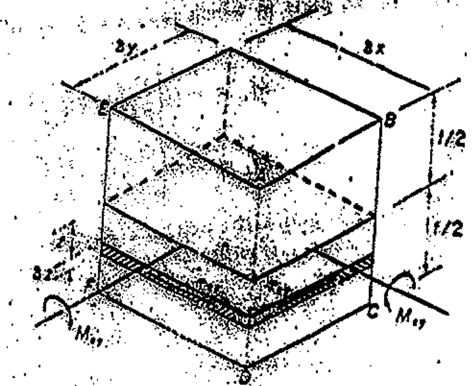


Fig. 5.6 Complementary shear stresses due to twisting moments M_{xy}.

and on the face $ADFE$

$$M_{xy}\,\delta x = -\int_{-t/2}^{t/2} \tau_{xy}\,\delta x\, z\, dz$$

giving

$$M_{xy} = -\int_{-t/2}^{t/2} \tau_{xy}\, z\, dz$$

or in terms of the shear strain γ_{xy} and modulus of rigidity G

$$M_{xy} = -G\int_{-t/2}^{t/2} \gamma_{xy}\, z\, dz \tag{5.12}$$

Referring to Eqs. (1.20) the shear strain γ_{xy} is given by

$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

We require, of course, to express γ_{xy} in terms of the deflection w of the plate; this may be accomplished as follows. An element taken through the thickness of the plate will suffer rotations equal to $\partial w/\partial x$ and $\partial w/\partial y$ in the xz and yz planes respectively. Considering the rotation of such an element in the xz plane as shown in Fig. 5.7 we see that the displacement u in the x direction of a point a distance z below the neutral plane is

$$u = -\frac{\partial w}{\partial x} z$$

[18]

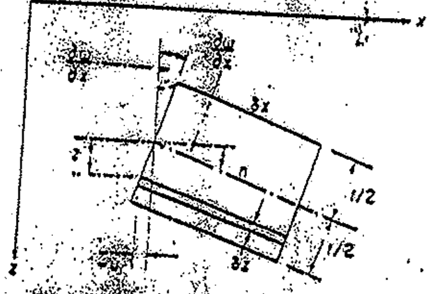


Fig. 5.7 Determination of shear strain γ_{xy}

Similarly the displacement v in the y direction is

$$v = -z\frac{\partial w}{\partial y}$$

Hence substituting for u and v in the expression for γ_{xy} we have

$$\gamma_{xy} = -2z\frac{\partial^2 w}{\partial x\,\partial y} \tag{5.13}$$

whence from Eq. (5.12)

$$M_{xy} = G\int_{-t/2}^{t/2} 2z^2\frac{\partial^2 w}{\partial x\,\partial y}\,dz$$

or

$$M_{xy} = \frac{Gt^3}{6}\frac{\partial^2 w}{\partial x\,\partial y}$$

Replacing G by the expression $E/2(1+\nu)$ established in Eq. (1.45) gives

$$M_{xy} = \frac{Et^3}{12(1+\nu)}\frac{\partial^2 w}{\partial x\,\partial y}$$

Multiplying numerator and denominator of this equation by the factor $(1-\nu)$ yields

$$M_{xy} = D(1-\nu)\frac{\partial^2 w}{\partial x\,\partial y} \tag{5.14}$$

Eqs. (5.7), (5.8) and (5.14) relate the bending and twisting moments to the plate deflection and are analogous to the bending moment-curvature relationship for a simple beam.

5.3 Plates subjected to a distributed transverse load

The relationships between bending and twisting moments and plate deflection are now employed in establishing the general differential equation for the solution of a thin rectangular plate supporting a distributed transverse load of intensity q per unit area (see Fig. 5.8). The distributed load may, in general, vary over the surface of the plate and is therefore a function of x and y. We assume, as in the preceding analysis,

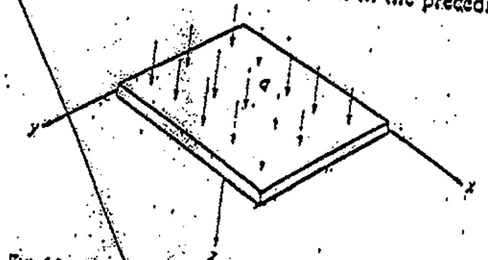


Fig. 5.8 Plate supporting a distributed transverse load.

that the middle plane of the plate is the neutral plane and that the plate deforms such that plane sections remain plane after bending. This latter assumption introduces an apparent inconsistency in the theory. For plane sections to remain plane the shear strains γ_{xz} and γ_{yz} must be zero. However the transverse load produces transverse shear forces (and therefore stresses) as shown in Fig. 5.9. We therefore assume that although $\gamma_{xz} = \tau_{xz}/G$ and $\gamma_{yz} = \tau_{yz}/G$ are negligible the corresponding shear forces

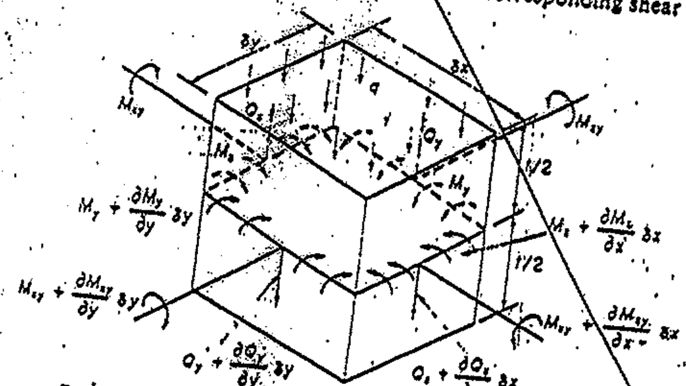


Fig. 5.9 Plate element subjected to bending, twisting and transverse loads.

[19]

To determine the column failing stress at the transition ...

Method: 1 Johnson - Euler Equation

Column failing stress,

$$F_c = F_{cs} - \frac{F_{cs}^2}{4\pi^2 E}\left[\frac{L'}{\rho}\right]^2 \longrightarrow (1)$$

Here $F_{cs} \rightarrow$ crippling stress.

Assume F_{cs} to occur at slenderness ratio, $\frac{L'}{\rho} = 0$.

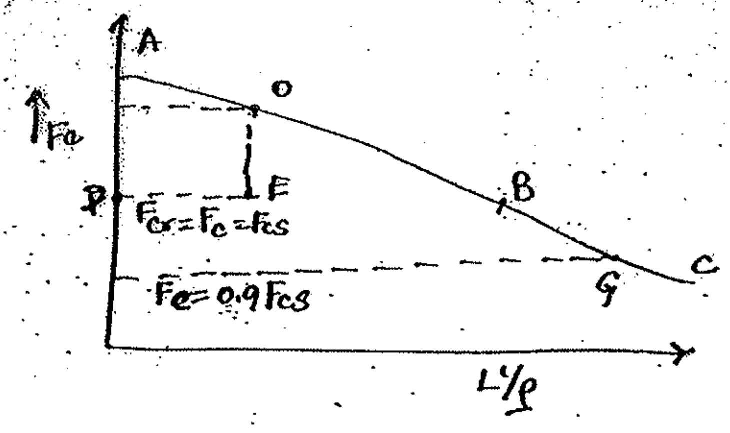
where $L' = \frac{L}{\sqrt{C}}$; where L = length of the column.

C = cross section area at the point where F_{cs} occurs.

Equation (1) gives a parabolic curve starting from F_{cs} at $\frac{L'}{\rho} = 0$. This method is quite simple and the additional calculation requires only the F_{cs} value of the column section and is obtained by various methods. Since F_{cs} is constant where $\frac{L'}{\rho} = 20$, so assumption of $\frac{L'}{\rho} = 20$ is slightly conservative.

Method: 2

The curve ABC is the Euler's column curve for a column with stable cross section for a given material



point "O" on the basic column curve by drawing a horizontal line through E value equivalent to F_{cy}. Draw a horizontal line starting from point D. Here D is at $F_c = F_{cs}$ and "E" point is found by vertical projection line from "O".

* Locate point "F" at $F_c = 0.9 F_{cs}$ and draw a horizontal line. Connect point E & G with a straight line. The line "EG" will give column failing stress for the value L'/ρ.

* This method requires buckling stress and column curve for of stable section. Here more graphical representations and calculations are needed.

Method: III Parabolic approximations.

The widely used method to represent the column strength in the transition range by using a parabolic approximation. It is

$$\frac{F_c}{F_{cs}} = 1 - \left[\left(1 - \frac{F_{cr}}{F_{cs}}\right)\left(\frac{F_{cr}}{F_E}\right)\right] \rightarrow (A)$$

Here F_{cs} = crippling stress.

F_{cr} = ~~crip~~ Buckling stress

F_c = column failure stress

F_E = Euler's column stress for a particular column.

$$F_E = \frac{\pi^2 E}{(L/\rho)^2}$$

This eqn (A) is applied for $F_c > F_{cr}$.

In case of $F_{cr} > F_{PL}$; then use F_{PL} instead of F_{cr} in eqn (A) where F_{PL} is propotional limit stress.

Problem 11.1 Peery Azar Page 351-353

The sheet stringer panel shown in Figure 11.28 is loaded in compression by means of rigid members. The sheet is assumed to be simply supported at the loaded ends and at the rivet lines. The sheet is assumed to be free at the sides. Each stringer has an area of 645.16 mm^2. Assume E = 71.157 GPa for the sheet and stringers. Find the total compressive load P:

(a) When the sheet first buckles.

(b) When the stringer stress σ_c is 69 $\frac{N}{mm^2}$ or 69 MPa.

(c) When the stringer stress σ_c is 207.34 $\frac{N}{mm^2}$ or 207.34 MPa.

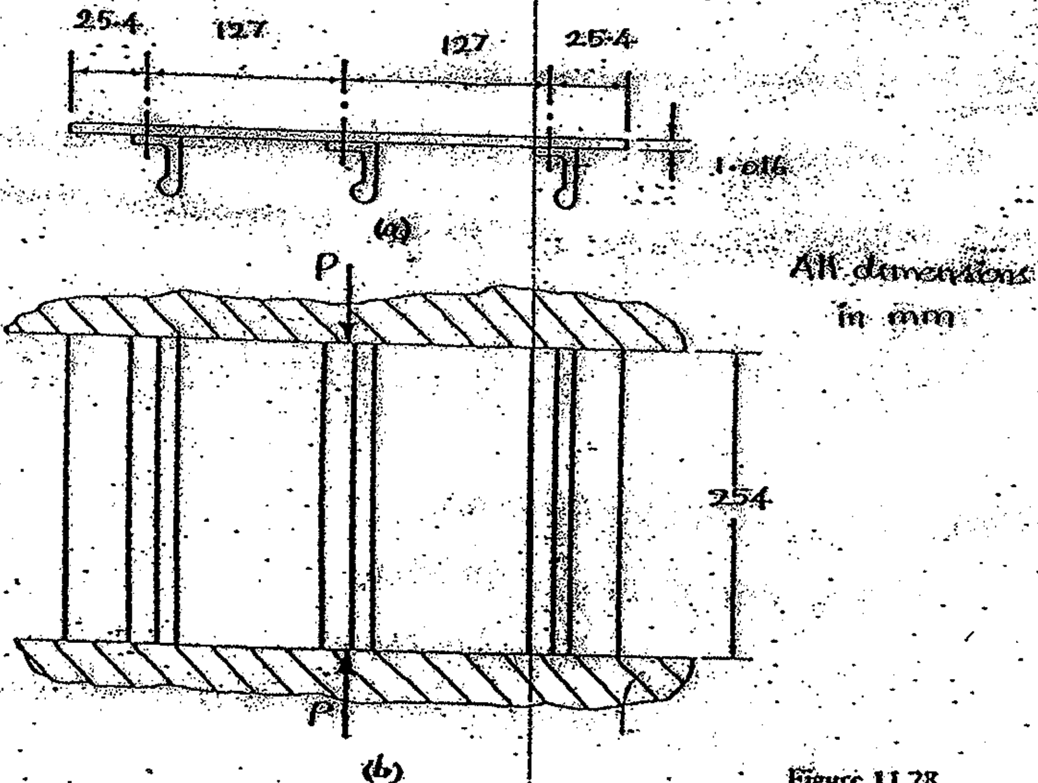


Figure 11.28

Solution:

(a) The sheet between the stringers is simply supported on all four edges and has dimensions of $a = 254$ mm, b = 127 mm and t = 1.016 mm. For $\frac{a}{b} = 2.0$, a buckling coefficient value of $K = 3.62$ is assumed.

The buckling stress is $\sigma_c = KE\left(\frac{t}{b}\right)^2 = 3.62 \times 71.187 \times 10^3 \times \left(\frac{1.016}{127}\right)^2 = 16.51828 \frac{N}{mm^2}$

The edge of the sheet has dimensions of $a = 25.4$ mm, $b = 254$ mm. It is simply supported on three edges and free on the fourth edge. a buckling coefficient value of $K = 0.385$ is assumed.

The buckling stress is $\sigma_c = KE\left(\frac{t}{b}\right)^2 = 0.385 \times 71.187 \times 10^3 \times \left(\frac{1.016}{25.4}\right)^2 = 43.851192 \frac{N}{mm^2}$

The sheet therefore buckles initially between the stringers. The total area of the sheet is assumed to be effective before buckling occurs. The buckling of a flat sheet in compression is a gradual process, and the load does not drop appreciably when buckling occurs. The load is therefore calculated as follows:

$A =$ area of stringers + area of sheet

$A = (3 \text{ stringers} \times 64.516 \text{ mm}^2 \text{ area}) + (\text{total width of sheet} \times \text{thickness of sheet})$

$A = (3 \times 64.516) + \{(25.4 + 127 + 127 + 25.4) \times 1.016\}$

$A = 503.2248\ mm^2$

$P = \sigma_c A = 16.51828 \times 503.2248 = 8{,}312.408149\ N = 8.312408149\ kN$

(b) The effective sheet widths are obtained from $w_e = 0.85t\sqrt{\frac{E}{\sigma_c}}$ and $w_{et} = 0.60t\sqrt{\frac{E}{\sigma_c}}$.

$$w_e = 0.85t\sqrt{\frac{E}{\sigma_c}} = 0.85 \times 1.016 \times \sqrt{\frac{71.187 \times 10^3}{69}} = 27.73885\ mm$$

$$w_{et} = 0.60t\sqrt{\frac{E}{\sigma_c}} = 0.60 \times 1.016 \times \sqrt{\frac{71.187 \times 10^3}{69}} = 19.58\ mm$$

$\sigma = \frac{K\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{a}\right)^2$

four SSB = 3.62

3SSB − 1 free = 0.385

The effective sheet area is

$$A_1 = (4w_e + 2w_{et})t = \{(4\times 27.73885) + (2\times 19.58)\}\times 1.016 = 152.517\ mm^2$$

$$P = \sigma_c A = 69\times(\text{stringer area + sheet effective area})$$

$$P = 69\times\{(3\times 64.516) + 152.517\} = 23,878.485\ N = 23.878.85\ kN$$

(c) The solution is similar to that of part (b).

$$w_e = 0.85t\sqrt{\frac{E}{\sigma_c}} = 0.85\times 1.016\times\sqrt{\frac{[illegible]}{207.34}} = 16.00\ mm$$

$$w_{et} = 0.60t\sqrt{\frac{E}{\sigma_c}} = 0.60\times 1.016\times\sqrt{\frac{71.787\times 10^3}{207.34}} = 11.2954\ mm$$

The effective sheet area is

$$A_1 = (4w_e + 2w_{et})t = \{(4\times 16) + (2\times 11.2954)\}\times 1.016 = 86.5908\ mm^2$$

$$P = \sigma_c A = 69\times(\text{stringer area + sheet effective area})$$

$$P = 207.34\times\{(3\times 64.516) + 86.5908\} = 58,083978\ N = 58.083978\ kN$$

Gerard's method to determine crippling stress

Gerard's extensive experimental investigation lead to the formation of three semi-empirical equations for determining the crippling stresses in various shapes of structural members. This method is the generalization of Needham's method. The Gerard's method recognizes the effect of distortion of the free unloaded edges upon the failing strength of the member section.

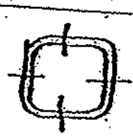
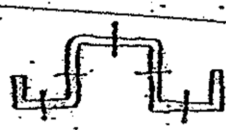
For sections with distorted unloaded edges such as tubes, V-groove plates, angles, stiffened panels and multi-corner sections	$\sigma_{crippling} = 0.56\sigma_{cy}\left[\frac{gt^2}{A}\left(\frac{E_c}{\sigma_{cy}}\right)^{0.5}\right]^{0.40}$
For sections with straight unloaded edges such as plates, T, cruciform (+) and H-sections	$\sigma_{crippling} = 0.67\sigma_{cy}\left[\frac{gt^2}{A}\left(\frac{E_c}{\sigma_{cy}}\right)^{0.5}\right]^{0.85}$
For sections such as two-corner sections, J, Z and C (channel) sections	$\sigma_{crippling} = 3.2\sigma_{cy}\left[\frac{t^2}{A}\left(\frac{E_c}{\sigma_{cy}}\right)^{1/3}\right]^{0.75}$

where

A = cross-sectional area

$g = \begin{Bmatrix}\text{the number of flanges which} \\ \text{make up the section}\end{Bmatrix} + \begin{Bmatrix}\text{number of cuts required to divide} \\ \text{the section into a number of flanges}\end{Bmatrix}$

f = flanges, c = cuts

Basic angle section g = 2f + 0c = 2 	T-section g = 3f + 0c = 3 
Square tube cross-section g = 8f + 4c = 12 	H-section or I-section g = 6f + 1c = 7 
Cruciform section g = 4f + 0c = 4 	Hat section g = 12f + 5c = 17 

Needham's method or the Angle method to determine crippling stress

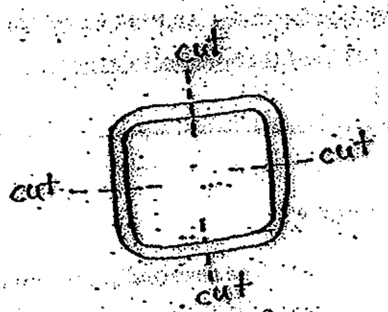
In this method, the structural member is divided into equal or unequal angles. The crippling failure strength of the member can be determined by summing the crippling stress of each angle element that makes up the total section. The crippling failure strength of a single angle unit is given by

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma_{crippling} = \frac{K_e\left(E_c\sigma_{cy}\right)^{0.5}}{\left(\frac{b'}{t}\right)^{0.75}}$$

where

σ_c	= crippling failure stress
E_c	= compressive Young's modulus
σ_{cy}	= compressive yield stress
$\frac{b'}{t}$	$= \frac{a+b}{2t}$
K_e	= 0.316 for two edges free = 0.342 for one edge free = 0.366 for no edges free



The total crippling stress for the entire member is

$$\left(\sigma_{crippling}\right)_{total\ section} = \frac{\sum_{i=1}^{n}\left(\sigma_{crippling}\right)_i A_i}{\sum_{i=1}^{n} A_i}$$

$$\sigma_{crippling} = \frac{K_e(E\sigma)}{b/t}$$

Effective width

Figure 1 shows a portion of a thin panel to which stiffeners are bonded, with the stiffeners spaced a distance b apart. The stiffened panel is shown supporting a compressive load. The stiffeners act to divide the panel into narrow plates of width b and length a, where a is the distance between the end supports of the panel. Taken individually, these plates have a higher aspect ratio than the panel itself, so the buckling stress is increased. Assume that the stiffeners apply a hinged support (or simply support) to the panels.

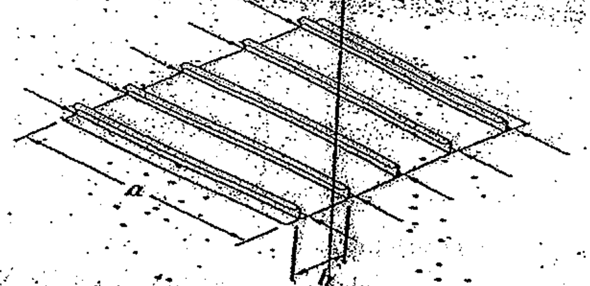


Figure 12.9.5 Stiffened panel in compression.

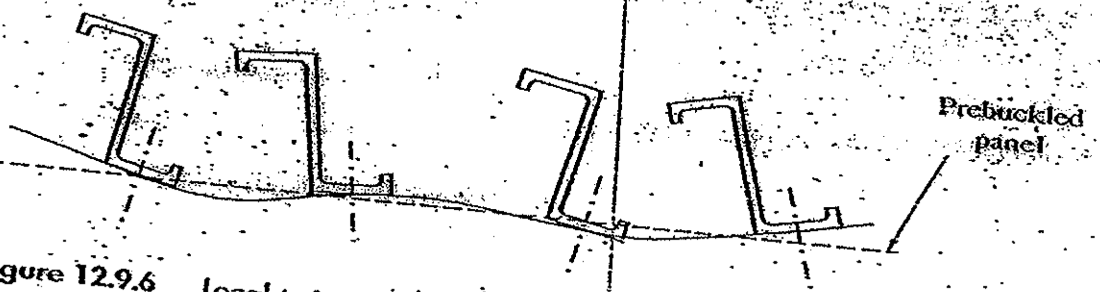


Figure 12.9.6 Local twist of stiffeners that apply simply-supported edge constraint to the skin subregions.

Inter-fastener buckling

Inter-fastener buckling may occur if the fastener spacing is relatively large compared to the skin thickness. In such a case, the skin wrinkles between the fasteners, but the stiffener flange remains essentially straight.

Sheet wrinkling failure

If the fastener pitch is small enough to prevent inter-fastener buckling, there is the possibility of sheet wrinkling failure. In this longer-range buckling mode, the skin does not bow away from the stiffener between the fasteners, but tends to force the stiffener flange to deform with it. This, in turn, induces stress in the stiffener web, possibly leading to local crippling of the stiffener.

I.F.B. / SWF

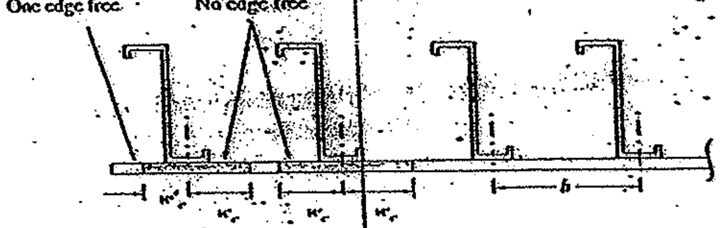


Figure 12.9.8 Cross section of a stiffened panel, the left end of which is free.

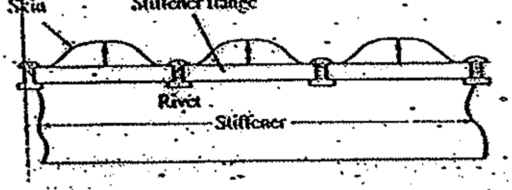


Figure 12.9.9 Interfastener (in this case interrivet) buckling.

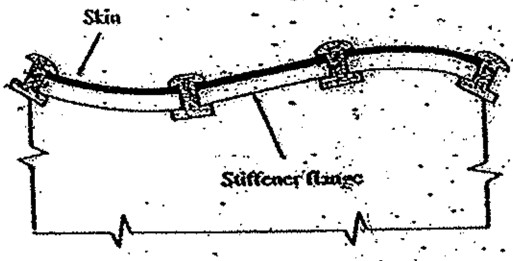


Figure 12.9.10 Wrinkling failure.

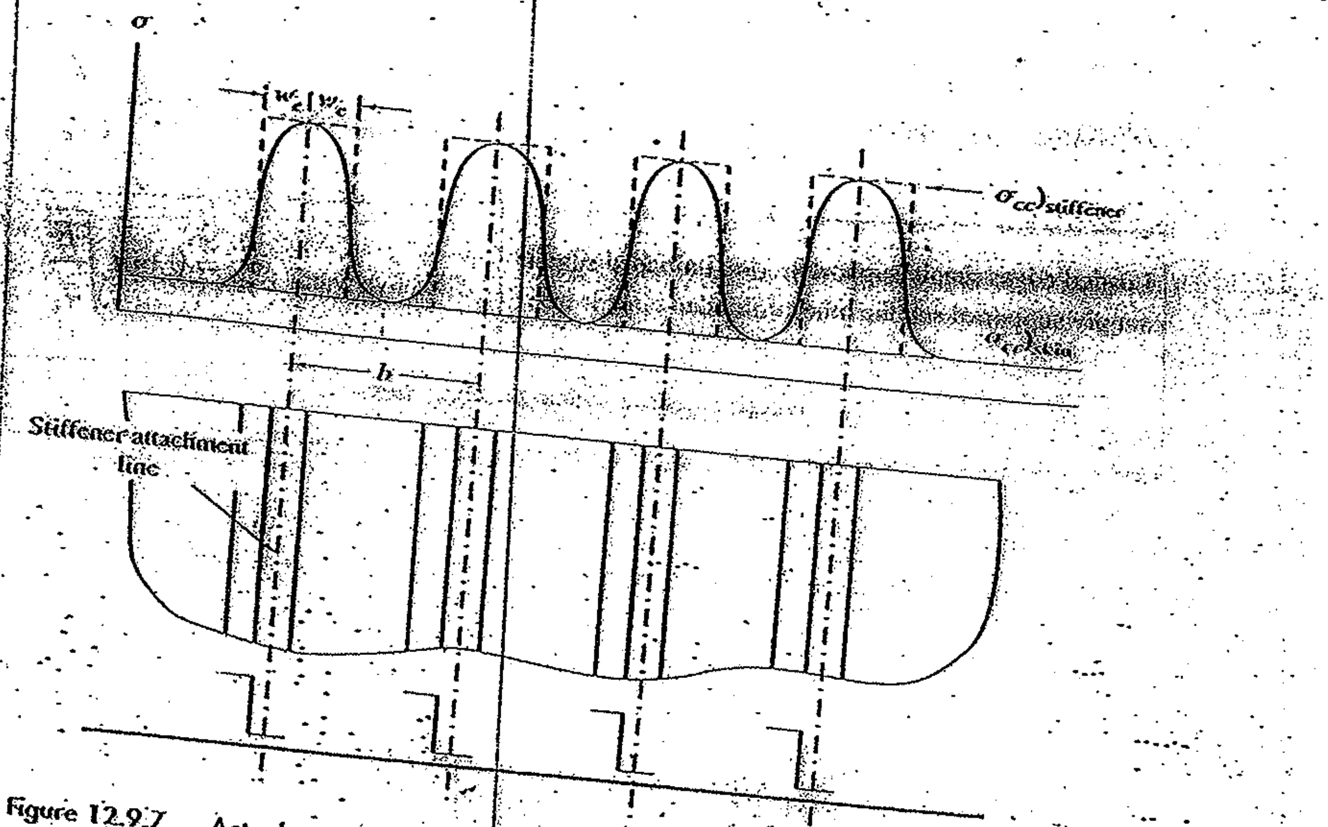
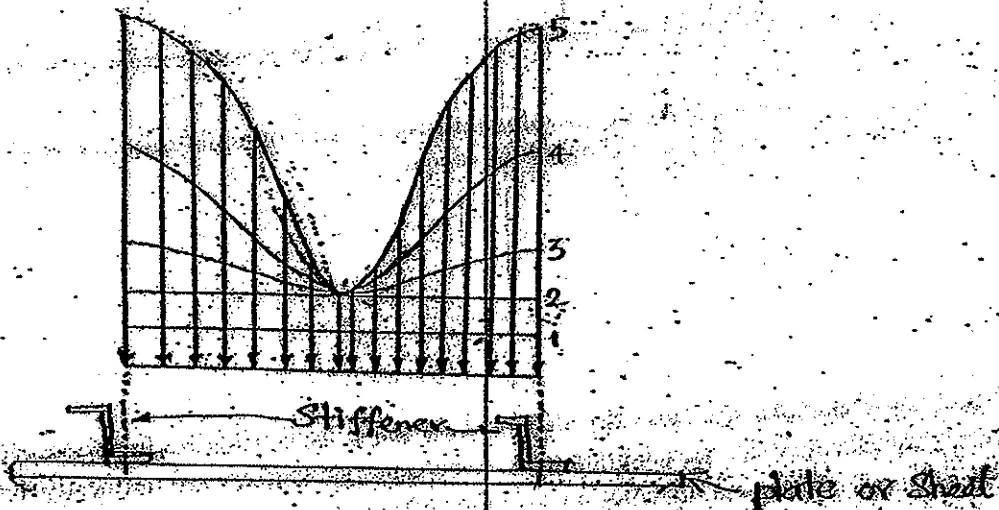


Figure 12.9.7 Actual versus equivalent stress distribution in stiffened panel structure after buckling of the skin.

At low load levels, both the skin and the stiffeners are active in distributing the stress. If the applied load is smaller than the buckling load, the compressive stresses are uniformly distributed as shown in figure 2. The stress distribution over the width of the plate is indicated in Figure 2 by lines 1 and 2. Line 2 indicates the stress at the initial buckling (or initiation of buckling). As the compressive stress increases, the stress distribution is indicated by lines 3, 4 and 5. The skin buckles elastically, and any increase in load will be carried mainly by the stiffeners and the adjacent plate material, as illustrated in figure 3. The sinusoidal-like curve is only suggestive of the complex nature of the post-buckled stress distribution in the skin and the stiffeners. That distribution is replaced by the uniform stress distribution (dotted line) extending across each stiffener and into the adjacent skins by a distance w_e called the effective width of the plate.

All edges simply supported	Effective width $w_e = 0.85t\sqrt{\dfrac{E}{\sigma_c}}$
Loaded edges simply supported, one unloaded edge is simply supported and the other unloaded edge is free	Effective width $w_{e1} = 0.60t\sqrt{\dfrac{E}{\sigma_c}}$
where σ_c is the critical buckling stress	



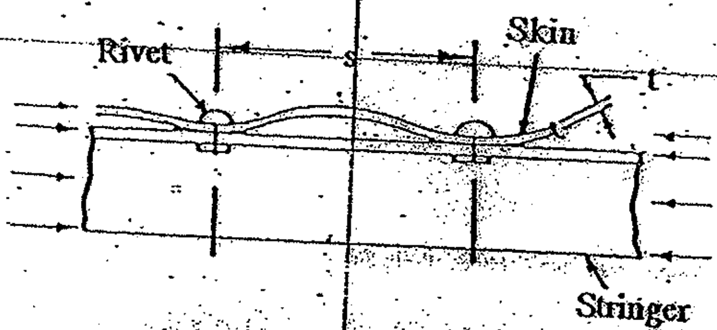
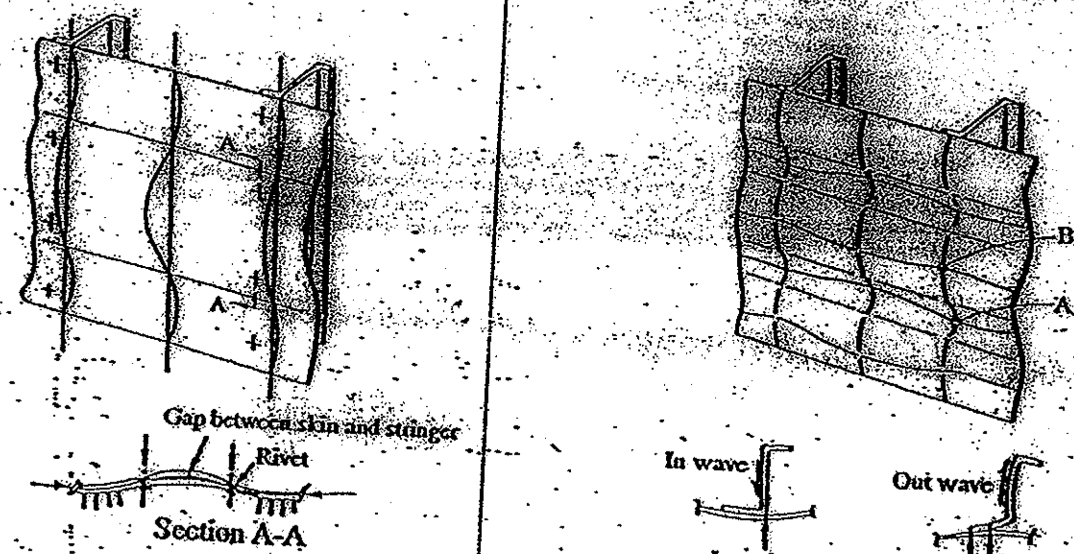


Fig. 14.3.1 Inter-rivet Buckling



(e) Inter-Rivet Buckling

(f) Skin Wrinkling

Fig. 14.4.1 Failure Modes of a Skin-Stringer Panel

FIGURE 8 Sheet inter-rivet buckling. (From Ref. 2.)

4.1. Inter-Rivet Buckling

[illegible] the sheet can buckle as a column between rivets [illegible] buckling check of a riveted joint, as shown in Figure 8, use the buckling formula [2]:

$$F_{ir} = KE\left(\frac{t}{s}\right)^2 \quad (1)$$

where

F_{ir} = buckling stress (psi)
t = thickness of sheet (in.)
b = [illegible] (in.)
s = [illegible] (in.)
E = [illegible] (psi)
[illegible] (in.)

and K is a factor [illegible] from Figure 9.

Note that this formula is based on a Poisson's ratio of 0.3 and is for elastic buckling only. Since the value varies from 0.28 to 0.35 for all metals, a value of 0.3 should be satisfactory for most calculations. Euler's formula can be used for more exact calculations [2]. A modification of Eq. (1) using the tangent modulus E_t must be used for plastic buckling.

[And also Refer Megson and Peery Book]

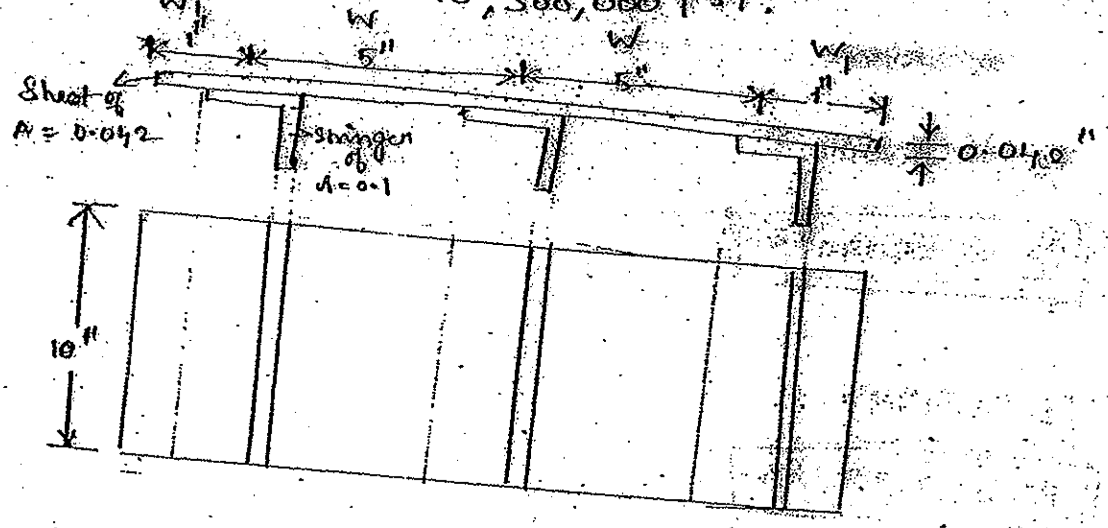
— [illegible]

of rigid mem. The sheet may be SS on all ends and the rivet lines they to be free at the sides. Each stringer has an area of 0.1 inch². Find the total compressive load for foll. condn.

i) when the sheet buckles first

ii) when the stringer stress is 10,000 psi = Fc

Assume E = 10,300,000 PSI.



i) $a = 10''$

$b = 5''$

$t = 0.042''$

$\frac{a}{b} = 2$; $K = 3.62$

$F_c = KE\left(\frac{t}{b}\right)^2$

$F_c = 2630.9$ PSI

33S+1F

$\therefore K = 0.385$

$F_c = 279.805$ PSI

$$A = A_{St} + A_{sheet}$$

$$= 3 \times 0.1 + 12 \times 0.042$$

$$A = 0.80 \text{ in}^2$$

$$P_c = F_c \times A$$

$$= \cancel{10{,}000 \times 0.8}$$

$$\cancel{= 8000}$$

$$\therefore \quad F_c = 2680.9$$

$$\boxed{P_c = 2104.72}$$

$$F_c = 279.8$$

$$\boxed{P_c = 228.84}$$

$F_c = 10{,}000$ Psi

$$W = t \times 0.85 \sqrt{\frac{E}{F_c}}$$

$$= 1.145 \text{ in.}$$

$$W_1 = 0.060 \times t \sqrt{\frac{E}{F_c}}$$

$$= 0.08$$

$$A_1 = (4W + 2W_1)t = 4.74 \times t \text{ in}^2 = 0.19908 \text{ in}^2$$

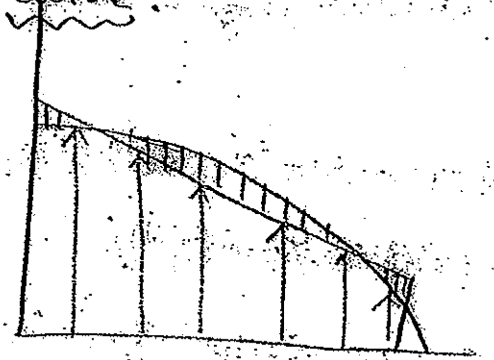
$$A = A_{St} + A_1 = 0.3 + \overset{0.19908}{\cancel{1.774}} = \cancel{5.04 \text{ in}^2}\ 0.49908 \text{ in}^2$$

Plates retain some of the capacity to carry loads even though when the portion of the plate has buckled infact the ultimate load is not reached untill the stress in the majority of the plate reaches the elastic limit. Gerald proposes semi-emprical soln. for flat plate having SS on all the edges.

Schrenk's curve.

$$\sigma_{cr} = \frac{\pi^2 k}{12(1-\nu^2)}\left(t/b\right)^2$$

Chord



Span

The load distribution on an idealised wing can be approximatly found out by making use of Schrenk's curve. Here a trapezoidal planform is considered and a quadrant of an ellipse whose area is equal to the area of the trapezium is superimposed on the trapezoidal planform. A curve which is obtained by joining all the pts. bet' the trapezoidal planform and

Schrenk's curve and the load obtained is air load on the wing. This method is valid for wings without any sweep.

For the wings with sweep we need to use lifting line theory to find out the airload distribution.

$$\frac{\overline{\sigma_f}}{\overline{\sigma_{cy}}} = \beta_g \left[g \, t_{sk} \, t_{st} \left(\frac{E}{\overline{\sigma_y}} \right)^{1/2} \right]^m$$

where

m & β_g are two consts which are to be determined experimentally.

$\overline{\sigma_f}$ is the failure stress of the panel.

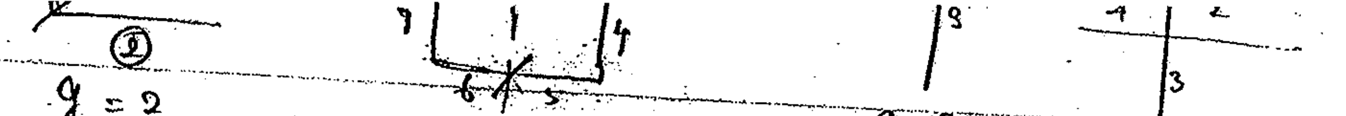
σ_{cy} - yeild stress

$\overline{\sigma_{cy}}$ - Avg. yeild stress

t_{sk} - thickness of the skin

t_{st} - Thickness of the stringer

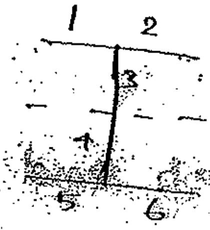
g - no. of cuts required for the panels to make it into a flange.



$g = 2$ $g = 4$ cuts $g = 3$ $g = 4$

$= 8$ flang

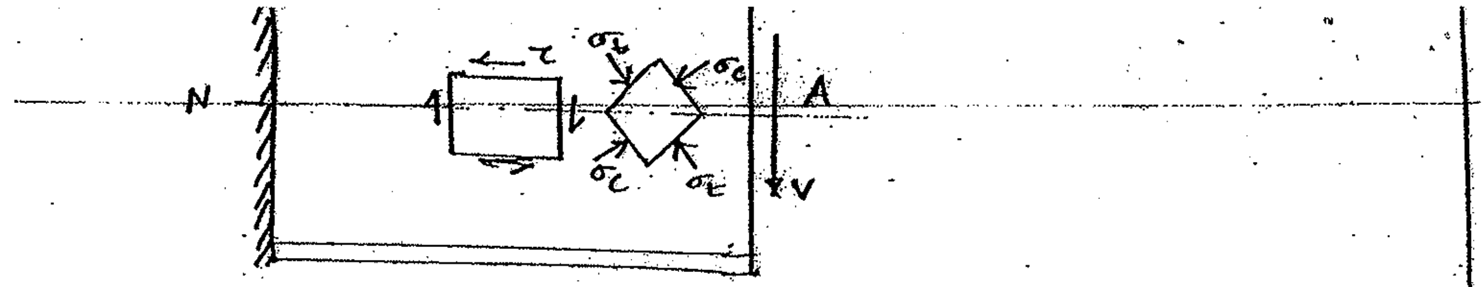
$\therefore g = 12$



$g = \frac{1}{\frac{6}{7}}$

* Pure tension field wings (Wagner wing)

- The ultimate strength of a thin web in shear is much better than initial buckling strength. In the case of struc. members which were not exposed to air stream such as spars, webs will resist shear and they would be permitted to undergo wrinkling which is an another form of buckling and a small fraction of their ultimate loads. In order to describe the manner in which the loads are resisted in form of shear by the webs after buckling has taken place. It is convenient to consider pure tension in the beam, as a means of explanation and such a beam is called as a tension field beam (37)



Consider a beam which is having a thick web and it is under the action of both shear and bending.

Normal stress along the NA, $\sigma_x = \frac{My}{I}$

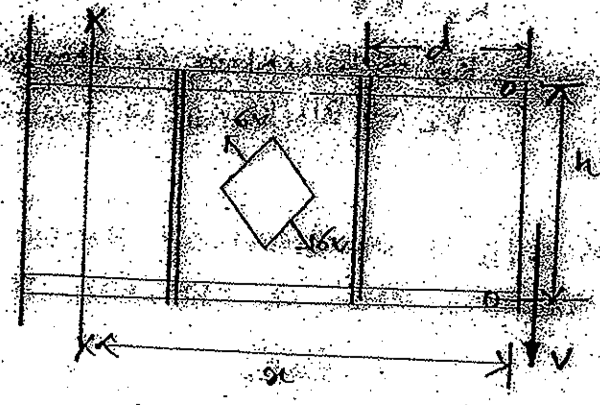
$$\tau = \frac{VQ}{I \cdot b}$$

Consider the element of the web along the NA so that bending stresses are absent in that case $\sigma_t = \sigma_c = \sigma_s$ or τ

When there are no normal stresses acting on the web, the web will be subjected to pure shear and these stresses can be found out using mohr's circle and this pt. these stresses act at 45° to the dirn of the applied shear stress. Now assume that the web becomes thinner. The normal stresses will now have another component and that will be in compressive nature. And it will cause the plate to buckle.

[3.8]

of applying tensile forces at the flanges.
These tensile forces will try to pull the 2 flanges together making it necessary to introduce vertical members to contract this bending.



$$\sigma_t = \frac{2V}{ht} \cdot \frac{1}{\sin\alpha}$$

$$F_t = \frac{V \cdot x}{h} - \frac{V}{2}\cot\alpha$$

$$F_c = -\frac{Vx}{h} - \frac{V}{2}\cot\alpha$$

where
- V - shear load
- h - effective sheet width
- t - web thickness
- d - stringer spacing
- α - angle of buckle

~~Find the buckling stress and margin of safety~~
and also check the same for the critical Panel

Area of each Stringer is equal to $2cm^2$

$K_c = 4$

$K_s = 5.8$

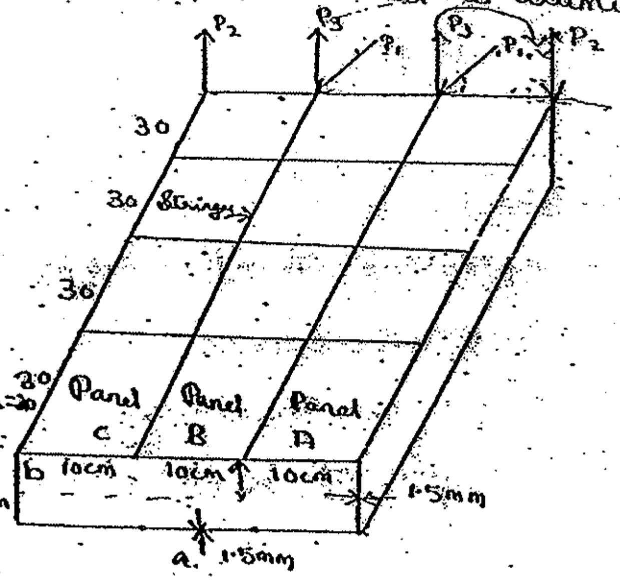
$K_s \Rightarrow$ Buckling co-efficient for Shear

$K_c \Rightarrow$ Buckling co-efficient for Compression

For $a/b = 3$ in bending since the panel is thin. Assume skin is effective in bending

[OR]

The fig shows a portion of cantilever wing composed of sheet, stiffness & ribs. The problem is to determine whether skin panels marked (A), (B), & (C) will buckle under the various given load cases. The sheet material is aluminium alloy 2024 T_3



i) When $P_1 = 10{,}000N$, $P_2 = 0$; $P_3 = 0$

ii) When $P_1 = 10{,}000N$; $P_2 = 5000N$; $P_3 = 0$

$$\sigma_{cr} = \frac{K_c \pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$$

$\times 20 \times 10^{5}\ N/cm^2$

$$= \frac{4\pi^2 \times (70\times10^5)}{12(1-0.3^2)}\left(\frac{0.15}{10}\right)^2$$

$= 5694\ N/cm^2$

$= 56.94\ MPa$

Since $\sigma_{cr} > (\sigma_c)_{P_1}$. The Panel will not buckle

$R_c \rightarrow$ Stress ratio

$$R_c = \frac{13.86}{56.94} = 0.2434$$

$$F.O.S = \frac{1}{R_c} = 4.07$$

Margin of Safety = F.O.S − 1

M.O.S = 3.07

1-03-05

Case (iv)

$P_1 = 10000\ N$; $P_2 = 5000\ N$; $P_3 = 0$

Due to P_2 which is Couple

Induced Shear due to couple

$T = 2A\tau t$

$\tau = \frac{T}{2At}$

$q = \tau \times t$

$\tau = q/t$

$= \frac{T}{2At}$

$$\tau = \frac{Torque}{2At} = \frac{5000\times30}{2\times30\times20\times0.15} = 833.33\ N/cm^2$$

$(\tau)_{P_2} = 8.33\ MPa$

Critical Shear Stress

$$\tau_{cr} = \frac{K_s \pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$$

$$= \frac{5.8\times\pi^2\times70\times10^5}{12(1-0.3^2)}\left(\frac{0.15}{10}\right)^2 = 82.56\ MPa$$

STRESS Ratio(R):

$$R = R_c + R_s^2$$

$$R_s = \frac{\tau}{\tau_{cr}} = \frac{8.33}{82.56} = 0.10089$$

$$R = 0.2434 + (0.10089)^2$$

$$= 0.253$$

Since $R<1$ the panel will not buckle.

$$M.O.S = \frac{2}{R_c + \sqrt{R_c^2 + 4R_s^2}} - 1 = \frac{2}{0.2434 + \sqrt{0.2434^2 + 4 \times 0.10089^2}} - 1$$

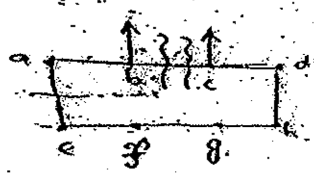
$$M.O.S = 2.574$$

CASE: 3

When $P_1 = 1000N$; $P_2 = 5000N$; $P_3 = 1000N$

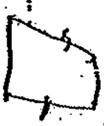
Comp. stress due to P

$(\sigma_c)_{P_1} = 13.86 MPa$ $\quad (\sigma_c)_{cr} = 56.94 MPa$



$(\tau_{P_2}) = 8.33 MPa$ $\quad (\tau_{cr})_{P_2} = 82.56 MPa$

in case 3, There will be shear flow due to load P_3



$$q = -\frac{V}{I} \Sigma y dA$$

$$= -\frac{2000}{2700} [\Sigma y dA]$$

$$= -0.7407 \Sigma y dA$$

Take a cut at 'bc'

$$q_{bc} = 0$$

$$q_{fg} = 0$$

1Pa

$q_{ba} = -0.74074\,[10\times2] = -14.81$ N/cm

$q_{ae} = -0.74074\,[10\times2] - 14.81 = -29.6$ N/cm

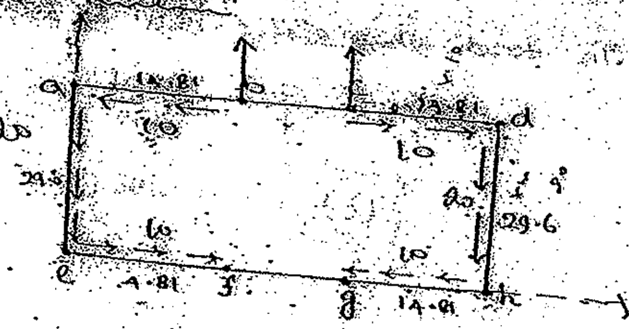
$q_{ef} = -29.6 - 0.74074\,[-10\times2] = -14.81$ N/cm

$q_{gh} = 0 - 0.74074\,[-10\times2] = 14.81$ N/cm

$q_{hd} = 14.81 - 0.74074\,[-10\times2] = 29.6$ N/cm

$q_{dc} = 29.6 - 0.74074\,[10\times2] = 14.81$ N/cm

Shear flow diagram



ΣM_e

$= -1000\times10 - 1000\times20 - 14.81\times10\times20 + 29.6\times20\times20 + 14.81\times10\times20$

$M_e = -18240$ N cm

Balanced moment $= 18240$ N cm

W.K.T according to BBT

$$M = 2Aq$$

$$q = \frac{M}{2A} = \frac{18240}{2\times30\times20} = 10.2 \text{ N/cm}$$

Balanced Shear flow Values

$q_{ba} = -14.81 + 10.2 = -4.61$ N/cm

$q_{ae} = -19.4$ N/cm

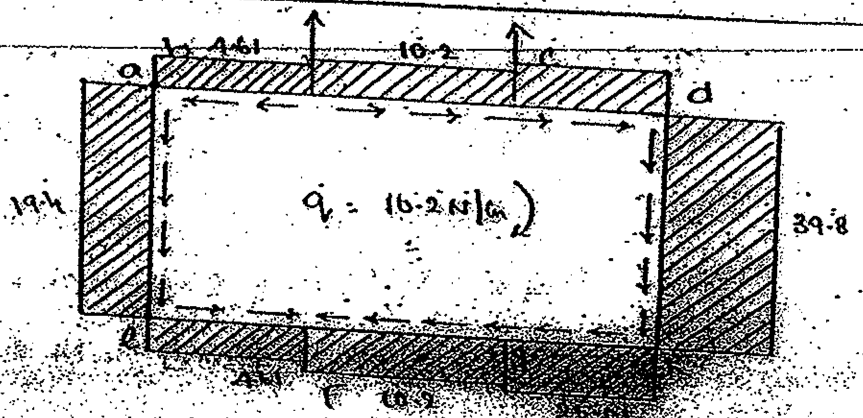
$q_{ef} = -4.61$ N/cm

$q_{fg} = 10.2$ N/cm

$q_{gh} = 25.01$ N/cm

$q_{hd} = 39.8$ N/cm

$q_{dc} = 25.01$ N/cm



To find shear stress due to load (P_3)

$$(\tau)_{P_3} = \frac{q}{t} = \frac{25.61}{0.15} = 1.67\ \text{MPa}$$

$$\boxed{(\tau)_{P_3} = 1.67\ \text{MPa}}$$

$$R_s = \frac{\tau}{\tau_{cr}} = \frac{(\tau)_{P_2} + (\tau)_{P_3}}{\tau_{cr}} = \frac{8.33 + 1.67}{82.56} = 0.1211$$

$$\boxed{R_s = 0.1211}$$

To find R_c

$$R_c = \frac{(\sigma_c)_{Tot}}{\sigma_{cr}} = \frac{(\sigma_c)_{P_1} + (\sigma_c)_{P_3}}{\sigma_{cr}}$$

$$(\sigma_c)_{P_3} = \frac{M \times y}{I} = \frac{2000 \times 180}{8700} \times 10 = 8.889\ \text{MPa}$$

$$R_c = \frac{13.86 + 8.889}{56.94} = 0.4$$

$$R = R_c + R_s^2 = 0.4 + 0.1211^2 = 0.414$$

Since $R < 1$ there is no buckling

$$M.O.S = \frac{2}{R_c + \sqrt{R_c^2 + 4R_s^2}} - 1 = \frac{2}{0.4 + \sqrt{0.4^2 + 4 \times 0.1211^2}} - 1$$

$$\boxed{\begin{array}{l} M.O.S = 1.3 \\ F.O.S = 2.3 \end{array}}$$